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NATURAL CONVECTIVE OSCILLATORY FLOW IN CYLINDRICAL ANNULI

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Abstract—Detailed quantitative information is presented concerning the characteristics of a natural convective oscillatory flow in horizontal, cylindrical annuli. This unusual, three-dimensional oscillatory flow is described by giving amplitude, period, and wave length data as well as data showing when this type of flow has its inception from a previously stable flow at lower Grashof numbers. Visualization of the air flow was achieved by the use of tobacco smoke. Annulus pressure was varied from 13·8 to 275·8 kN/m^2 while the temperature difference between the cylinders was varied from 2·8 to 55·5 deg C yielding a Grashof number (based on inner-cylinder diameter) range from 290 to $2{\cdot}7 \times 10^6$. Correlation equations are given to facilitate data presentation and to allow estimation of the inception of this oscillatory flow and its subsequent amplitude, period, and wave length.

NOMENCLATURE

- g, acceleration of gravity;
- H_i, amplitude of oscillatory flow at inner-cylinder radius [rad];
- H_0, amplitude of oscillatory flow at outer-cylinder radius [rad];
- L, gap width, $r_0 - r_i$;
- N_D, relative gap width $L/2r_i$;
- N_E, expansion number $\beta\Delta T$;
- N_{Gr}, Grashof number, $g\beta(2r_i)^3\,\Delta T/\nu^2$;
- N_H, amplitude Reynolds number, $\rho r_0 H_0 L/\mu Z$;
- N_W, wave length Reynolds number, $\rho r_i H_i W/\mu Z$;
- N_Z, period Reynolds number, $\rho r_i H_i L/\mu Z$;
- P, annulus pressure;
- r, radial coordinate;
- r_i, radius of inner cylinder;
- r_0, radius of outer cylinder;
- T, temperature;
- T_i, temperature of inner cylinder;
- T_m, mean temperature, $(T_i + T_0)/2$;
- T_0, temperature of outer cylinder;
- ΔT, temperature difference, $T_i - T_0$;
- W, wave length of oscillatory flow;
- Z, period of oscillatory flow.

Greek symbols

- β, coefficient of thermal expansion;
- μ, dynamic viscosity;
- ν, kinematic viscosity;
- ρ, density;
- θ, angular displacement, measured from upward vertical.

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1. INTRODUCTION

A RECENT investigation of natural convective flows in horizontal cylindrical annuli by the authors [1] revealed a rather unusual flow oscillation the characteristics of which had not been previously reported in the literature. This report was followed by articles [2, 3] which gave descriptions of similar flows under varying conditions. The description of the oscillatory flow pattern by Bishop and Carley [1] was restricted to two-dimensional observations. In a subsequent paper by Grigull and Hauf [2] another unusual flow pattern was described which had three-dimensional characteristics.

Although the work by Bishop and Carley was over the same range of variables as that of Grigull and Hauf, they did not observe the three-dimensional flows described by the latter authors. The contradictory results from these two papers are described in detail by Bishop [4]. In a later note Grigull and Hauf [5] indicate that they also saw the oscillating flows described by Bishop and Carley but did not report on them in their first paper. In a paper by Lis [3] experimental results are presented which describe apparently the same oscillating flow condition.

Contradictions in the descriptions of these flows prompted a study directed at obtaining more detailed information concerning the onset and characteristics of this oscillatory flow. The present investigation was undertaken to extend the Grashof number range with vacuum and pressurized conditions in the annulus and, in conjunction with this extension of range, to obtain precise information concerning the characteristics of the flow oscillation phenomena. The reporting of three-dimensional flows by Grigull and Hauf prompted a search into possible three-dimensional characteristics of the subject oscillating flow. This paper presents the results of this study.

2. FLOW VISUALIZATION APPARATUS

The apparatus used in this investigation consisted basically of a heated copper inner cylinder which was mounted concentrically within a cooled glass outer cylinder. The inner cylinder contained a freon liquid–vapor mixture so that when an electric current was passed through a nichrome heater contained within this cylinder a continuous freon vaporization–condensation cycle was obtained, maintaining an isothermal inner cylinder surface. Located on a plywood enclosure for the cylinders were seven 6400 rev/min blowers which produced a forced downward air draft over the glass outer cylinder, maintaining an isothermal outer cylinder surface. Temperatures on each cylinder surface were measured by the use of twelve thermocouples located at random positions on the surface. Flow visualization of the annular space was accomplished by transverse illumination of a single 0·635-cm wide cross-section plane with an intense light source passed through a water filter and by viewing this plane from a longitudinal position through a glass end plate upon which the cylinders were mounted. A single copper inner cylinder of 4·13 cm O.D. and three glass outer cylinders with inside diameters of 7·62, 10·16 and 15·24 cm were employed to yield relative gap widths, $L/2r_i$, of 0·423, 0·731, and 1·35 respectively.

For this investigation, the basic apparatus as briefly described above, and as described in detail in [1], was modified to permit three-dimensional viewing of the annulus and to allow variations in the annulus pressure. A twelve inch wide, semi-circular band of paint was removed from the outer cylinders, and one of the blowers was removed from the plywood enclosure to facilitate transverse observation of the annular space when it was illuminated from the longitudinal direction. Two vertical slits, with an adjustable longitudinal spacing, were provided in the plywood enclosure for transverse illumination of the annulus so that the flows in adjacent vertical planes could be studied simultaneously.

The apparatus was also modified to allow annulus pressure variations as shown schematically in Fig. 1. When varying the pressure, it was necessary to introduce the smoke into the annulus after the pressure had been brought to its final value so that the smoke would remain in the central longitudinal plane instead of being dispersed throughout the annulus and making visual observations impossible. In view of this fact a secondary smoke chamber was connected to the annulus through a throttling valve, and a vacuum pump and compressed air source were connected in parallel to both the annulus and the smoke chamber. A mercury manometer was used for reading pressure differentials between the smoke chamber and the annulus, and a bourdon tube pressure gauge, in parallel with a bourdon tube vacuum

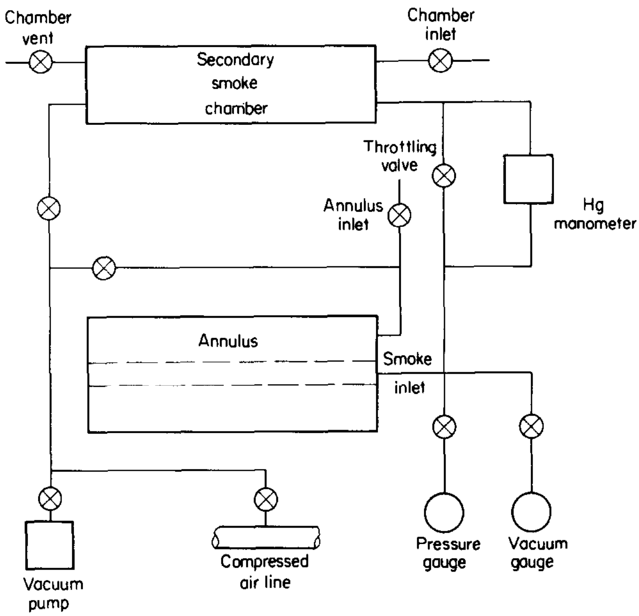

FIG. 1. Schematic diagram of apparatus.

gauge, was connected to the annulus for measuring pressures.

The data which were recorded in this investigation included inner and outer cylinder surface temperatures, photographs and motion pictures of the flow patterns, cylinder diameters, and annulus pressures. Table 1 gives the ranges of the variables for this study along with those of other investigators. The period, amplitude, and wave length were also recorded for those flow patterns which oscillated.

3. PROCEDURE

The procedures described below were utilized for each run. A direct current voltage was first applied to the heater and adjusted to give the desired temperature difference. The following method was then used to vary the annulus pressure and to introduce smoke into the annulus. First, smoke was blown into the secondary smoke chamber until a high smoke density was obtained. The annulus pressure was then brought to the desired value, and the pressure in the smoke chamber was adjusted to a value of about 5 mm of mercury higher than that in the annulus. By utilizing the throttling valve located between the chamber and the annulus, the amount of smoke allowed to enter the annulus could be accurately controlled with essentially no effect on the pressure.

For a given relative gap width both the temperature difference and the annulus pressure were varied over the ranges shown in Table 1. Neither the temperature difference nor the annulus pressure was consistently fixed and the other varied, but both were varied in a random manner.

In order to measure periods and amplitudes of the oscillatory flow attention was focused on that portion of the flow pattern termed the chimney. The chimney is defined as the upper region of the gap where the flows from both sides of the inner cylinder converge and rise toward the outer cylinder. The period of the oscillating flow patterns was measured using a stop watch, and a circular plexiglass plate (scribed in 5° increments) was mounted on the glass end plate concentric with the cylinders

Table 1. Ranges of variables for several investigators

Author	Fluid	D_0 (cm)	D_i (cm)	D_0/D_i	L (cm)	L/D_i	T_i (°C)	T_0 (°C)	$T_i - T_0$ (°C)	P (kN/m^2)	N_{Gr}(min)	N_{Gr}(max)
Liu,	air	19·1	2·54	7·5	8·28	3·25	—	—	1·28–80·3	atm	$6·6 \times 10^3$	$3·1 \times 10^8$
Muller and		19·1	5·08	3·75	7·01	1·38						
Landis [4]		19·1	7·61	2·50	5·75	0·75						
Tube length		19·1	12·7	1·50	3·20	0·25						
= 48·2 cm		19·1	16·5	1·15	1·30	0·077						
Bishop and	air	5·08	4·14	1·23	0·477	0·115	10–93·3	10–37·8	2·78–55·5	atm	$2·1 \times 10^4$	$3·2 \times 10^6$
Carley [1]		7·61	4·14	1·85	1·75	0·423						
Tube length		10·2	4·14	2·46	3·02	0·73						
= 91·5 cm		15·2	4·14	3·69	5·56	1·35						
Grigull	air	12	2	6	5	2·5	—	—	2·3–62·7	atm	$8·1 \times 10^3$	$1·6 \times 10^6$
and		12	4	3	4	1·0						
Hauf [2]		12	6	2	3	0·5						
Tube length		—	—	—	—	0·28						
= 50 cm		—	—	—	—	0·158						
Lis [3]	N_2 and	7·59	3·81	2	1·89	0·496	44·4–116·6	35·8–40	8·6–76·6	73·5–4200	—	—
Tube length	SF_6 and	11·39	3·81	3	3·79	0·995						
= 181 cm	mixtures	15·27	3·81	4	5·73	1·505						
Bishop	air	5·08	4·14	1·23	0·477	0·115	10–93·3	10–37·8	2·78–55·5	13·8–275·8	$2·9 \times 10^2$	$2·7 \times 10^6$
Carley and		7·61	4·14	1·85	1·75	0·423						
Powe		10·2	4·14	2·46	3·02	0·73						
Tube length		15·2	4·14	3·69	5·56	1·35						
= 91·5 cm												

to allow amplitude measurements. The amplitude was measured by simultaneously viewing the flow patterns and the plexiglass plate through a surveyor's transit which was located approximately 15 ft from the apparatus in order to reduce the parallax problem created by the 18-in separation of the glass end plate and the smoke plane. A three-dimensional effect producing a wave-like phenomena was also observed to occur for the oscillating flow patterns. When viewed in the transverse direction the chimney oscillated in such a manner as to give the appearance of a travelling wave. Measurement of the wave length was accomplished by adjusting the light planes until two adjacent chimneys were seen to oscillate in phase when viewed longitudinally and then determining the distance between the planes.

Photographs and motion pictures of the flow patterns were obtained by the method set forth in [1].

4. DISCUSSION OF RESULTS

In an earlier flow visualization study of the natural convection of air between horizontal concentric cylinders (Bishop and Carley [1]) we observed and photographed two basic types of flow; the crescent eddy pattern and the kidney-shaped eddy pattern. Figures 2 and 3 are representative photographs of these two patterns. In both patterns the fluid flows up along the inner-cylinder surface and down along the outer-cylinder surface at a relatively high speed when compared to the speed of the fluid in the central and major portion of the pattern. The primary differences between these two patterns are the shape of the central low-speed region and the size of the nearly stagnant region in the lower portion of the annulus. For a relative gap width of 1·35 and a Grashof number based on inner cylinder diameter of $1{\cdot}57 \times 10^5$ the normally stable kidney-shaped eddy pattern (Fig. 3) became unstable and began to oscillate tangentially (see Fig. 4); for all other relative gap widths and Grashof numbers achievable in the original apparatus the flow was stable.

The description presented in [1] of the oscillating flow condition is for a given r-θ plane and implies that the oscillations are longitudinally in phase, that is, specification of the amplitude and the period in any r–θ plane is sufficient to completely describe the oscillatory motion. This latter statement has since been found to be incorrect and will be discussed in detail in later portions of this section. The reader is referred to [1] for detailed qualitative descriptions of the basic patterns for various operating conditions of relative gap width and temperature difference between cylinders.

Grigull and Hauf [2] presented the results of a study similar to the one reported on in [1], and most of their photographs and qualitative descriptions of the flow characteristics agree with those given in [1]. However, they observed and photographed an unusual flow condition for "small" relative gap widths which is characterized by three-dimensional spirals in unsteady oscillating motion. After reviewing the results of their study, the question naturally arose as to whether or not the unstable oscillating flow described in our earlier paper [1] was in fact the same type of flow observed by Grigull and Hauf [2], even though it occurred for a considerable larger relative gap width.

In checking for possible three-dimensional effects in the unstable oscillating flow pattern reported on in [1], it was observed that the oscillations were in fact out of phase longitudinally. Although no flow occurred longitudinally, it was observed that different points along the chimney at a fixed radial position oscillated out of phase with each other forming a wave motion. The out-of-phaseness of the oscillations can best be described by referring to the schematic drawing of the chimney for various longitudinal positions shown in Fig. 5. In this figure the sections A–A through D–D show the chimney position for various longitudinal locations in the annulus. Grigull and Hauf [5], in the rebuttal to a discussion of their original paper, report having also observed this oscillating flow; but, since the influence of edge effects on the flow was unknown to them, they did not document their observations. Contrary to a statement by Grigull and Hauf that the wave

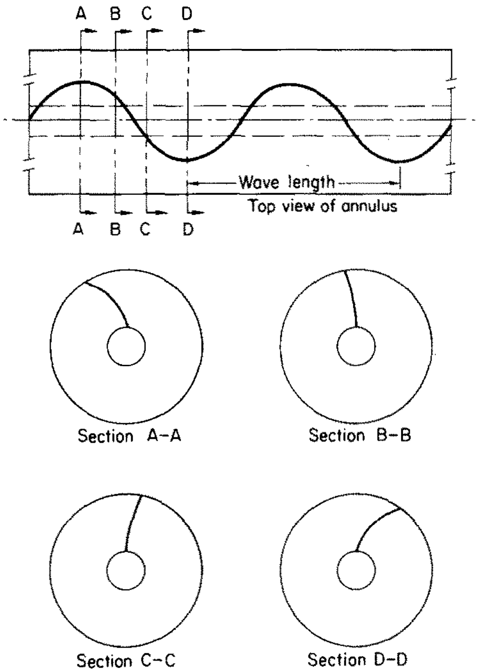

FIG. 5. Schematic of wave phenomena.

moved to and fro along the horizontal axis only in the middle of the tube, we observed this wave motion over the entire length of the annulus.

Before proceeding to a detailed description of the wave motion, edge effects will be briefly discussed. These effects are always present in natural convection flow studies of this type and should be considered in the evaluation of all flow conditions observed. Flow oscillations similar to those that are described in this paper have also been reported by Liu *et al.* [6], and by Lis [3], where in the first study the annulus length was 48·2 cm and in the latter was 181 cm. The annulus length in the study by Grigull and Hauf [2] was 50 cm and in the present work 91·5 cm. In each case the oscillations were found to start at approximately the same Grashof number even though there is a wide difference between the annulus lengths. Edge effects might cause the numerical values of wave length, period, and amplitude to vary somewhat as the annulus length is reduced much under 50 cm; however, there is little doubt that the oscillating flow condition is a true convective phenomenon and that documentation of the flow type should be pursued.

In order to completely describe the oscillating flow it was necessary to measure the wave length as well as the period and amplitude of the oscillation as the Grashof number and relative gap width were varied. The wave length is defined in the usual manner as the distance between two successive crests (see Fig. 5.) Since the chimney would become curved as an oscillation occurred, the tangential displacement of the chimney from the vertical centerline was a function of the radial coordinate. Immediately adjacent to the inner-cylinder surface the chimney displacement was independent of operating conditions and remained constant at about 5°. However, the displacement of the chimney in the vicinity of the outer-cylinder surface was a strong function of operating conditions, and it was here that the maximum displacement occurred. Although the maximum angular displacement of the chimney to either side of the vertical centerline varied somewhat, for given annulus conditions there appeared to be a constant average value of this displacement, and this average value was the same on both sides of the vertical centerline. The amplitude of an oscillation is thus defined as the average maximum angular displacement. The period of the oscillations is defined as the time required for the chimney to complete one to-and-fro movement.

The oscillating flow was observed to start for relative gap widths of 0·423, 0·731 and 1·35 at Grashof numbers of approximately $6{\cdot}6 \times 10^5$, $5{\cdot}6 \times 10^5$ and $1{\cdot}35 \times 10^5$ respectively. As the Grashof number was increased from these starting values the amplitude of the oscillations

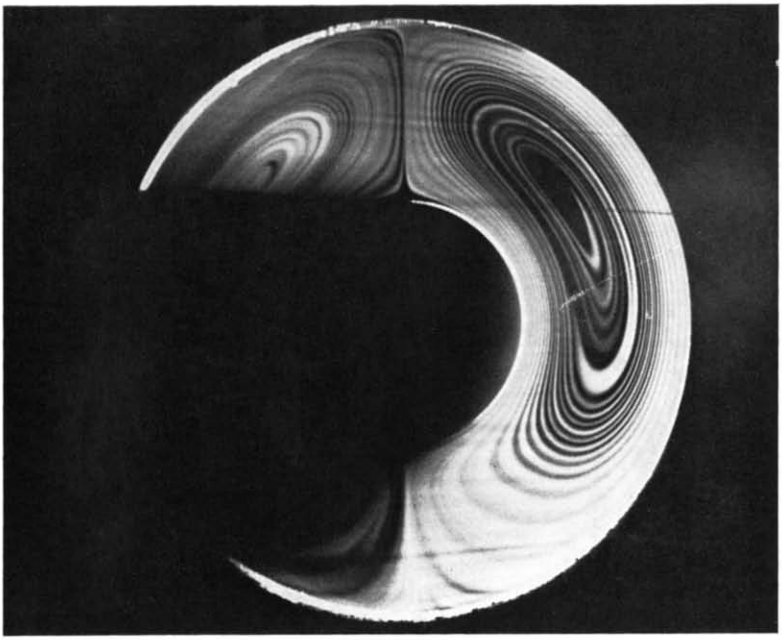

FIG. 2. Crescent eddy

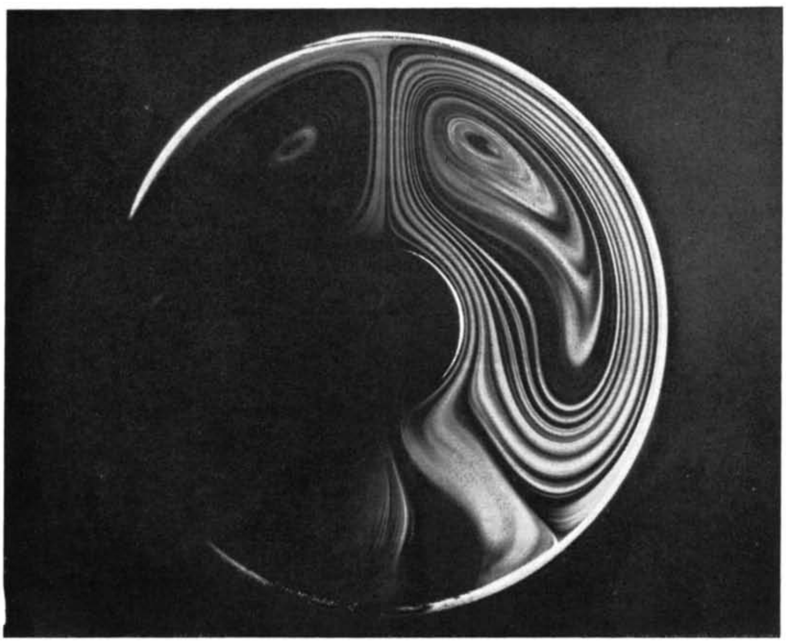

FIG. 3. Kidney-shaped eddy.

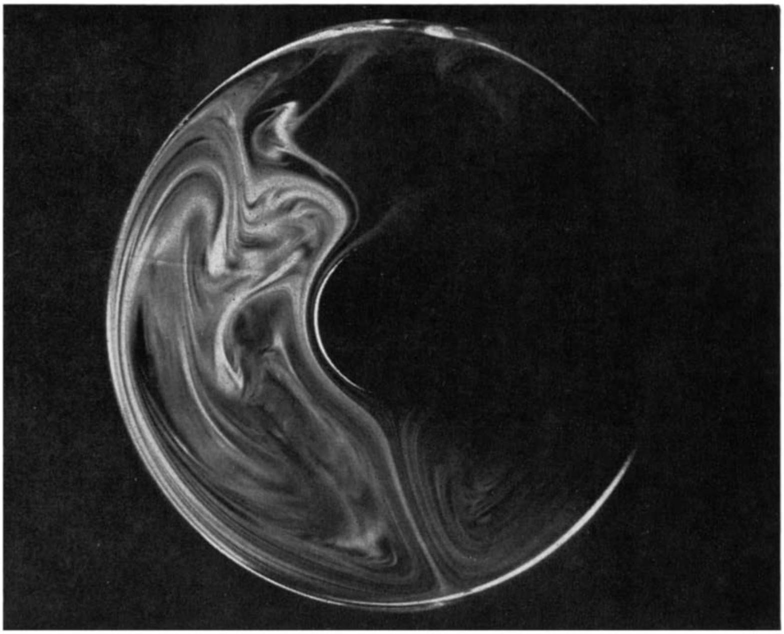

FIG. 4. Oscillatory flow.

increased and the period decreased. In no case did the flow revert to the stable crescent eddy or the stable kidney-shaped eddy types for maximum Grashof numbers of $2{\cdot}65 \times 10^6$, $2{\cdot}56 \times 10^6$ and $2{\cdot}66 \times 10^6$ as did the three-dimensional spiral flow observed by Grigull and Hauf [2].

Since the oscillating flow is an unstable form of the crescent eddy and kidney-shaped eddy patterns, it would be of great interest to be able to predict under what annulus operating conditions the flow would occur and the approximate magnitudes of the amplitude, period, and wave length of the oscillations. Prediction equations for these variables were sought utilizing appropriate dimensionless parameters determined from a dimensional analysis of the problem. Also, since over 500 data points were recorded, it was highly desirable to present the data in dimensionless form.

A dimensional analysis of the problem was performed using the following pertinent independent variables: gap thickness L, temperature difference between the cylinders ΔT, coefficient of thermal expansion β, acceleration of gravity g, fluid density ρ, and fluid viscosity μ. This analysis led to the following independent dimensionless groups:

$$N_{Gr} = g\beta\rho^2(2r_i)^3\Delta T/\mu^2, \tag{1}$$

$$N_D = L/2r_i, \tag{2}$$

$$N_E = \beta\Delta T, \tag{3}$$

where N_{Gr} is the Grashof number, N_E is the expansion number, and N_D is the relative gap width. Although these three dimensionless groups are the standard parameters for natural convection (e.g. see Ispen [8]), the effect of the expansion number is normally not considered to be significant, and it is usually neglected. The expansion number was retained in this case, however, since it was felt that expansion influences might be significant in this unsteady phenomenon. After extensive visual and photographic observation of the oscillatory flow, it was concluded that inertia forces and viscous forces were both highly significant. This conclusion led to the selection of Reynolds number as the primary dependent variable. This selection is supported by the successful prediction of the onset of another unstable flow in cylindrical annuli, (Powe [7]) using Reynolds number in association with the familiar vortex street phenomenon for cylinders in forced flow. Three Reynolds numbers were defined using velocities and dimensions which were characteristic of the three dependent variables whose correlation was desired; period, wave length, and amplitude. These Reynolds numbers are defined as follows:

$$N_Z = \rho H_i r_i L/\mu Z \tag{4}$$

$$N_H = \rho H_0 r_0 L/\mu Z, \tag{5}$$

$$N_W = \rho H_i r_i W/\mu Z, \tag{6}$$

where in each case the characteristic velocity, Hr/Z, is selected as that of the oscillating flow at either the inner or outer cylinder radius.

Using the period Reynolds number as the dependent variable, a correlation equation was sought utilizing only the relative gap width and the Grashof number as independent variables. This approach was found to be highly successful, and standard least squares techniques yielded the following equation:

$$N_Z = 0{\cdot}00586\, N_D^{\,0{\cdot}537}\, N_{Gr}^{\,0{\cdot}442}; \tag{7}$$

which predicts the period Reynolds number of the oscillating flow for all three relative gap-width configurations and for all values of temperature difference and annulus pressure with an average error of 4·8 per cent. The period Reynolds number is plotted vs. the Grashof number in Fig. 6 for each relative gap in order to show the specific effects of Grashof number and relative gap width on this Reynolds number. The success in correlating the data for the period of the oscillating flow using a Reynolds number led to a search for a similar correlation for the wave length data. This search was not quite as successful. Correlation was not possible unless the expansion number was included along with the Grashof number and relative gap

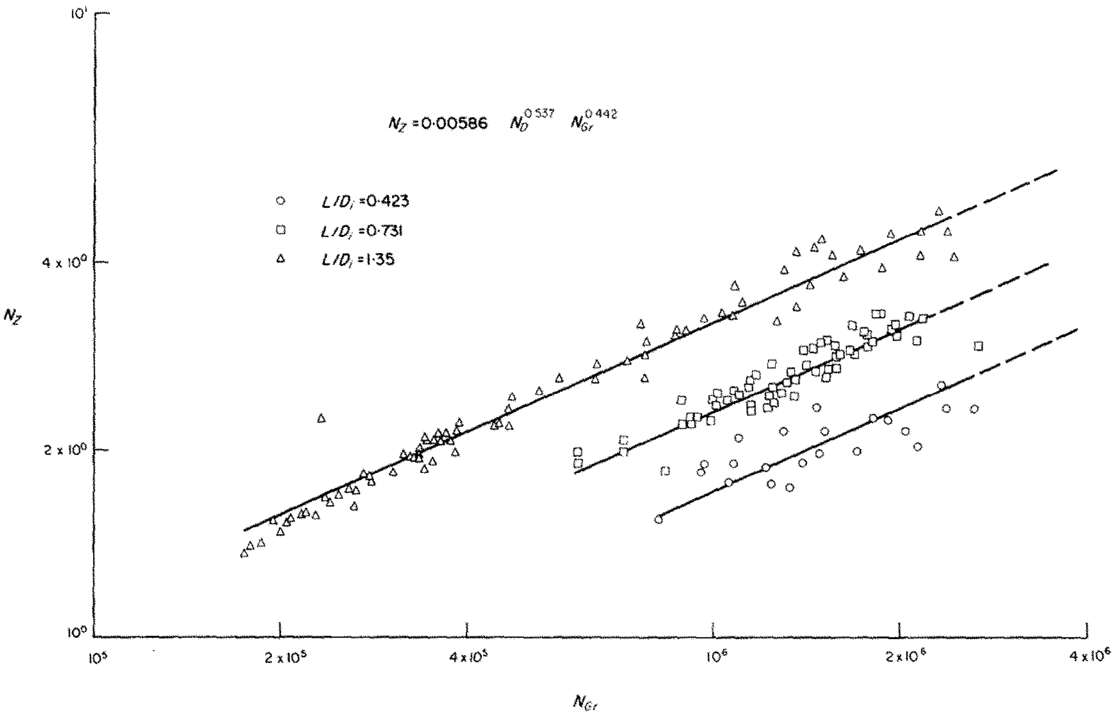

FIG. 6. Period correlation.

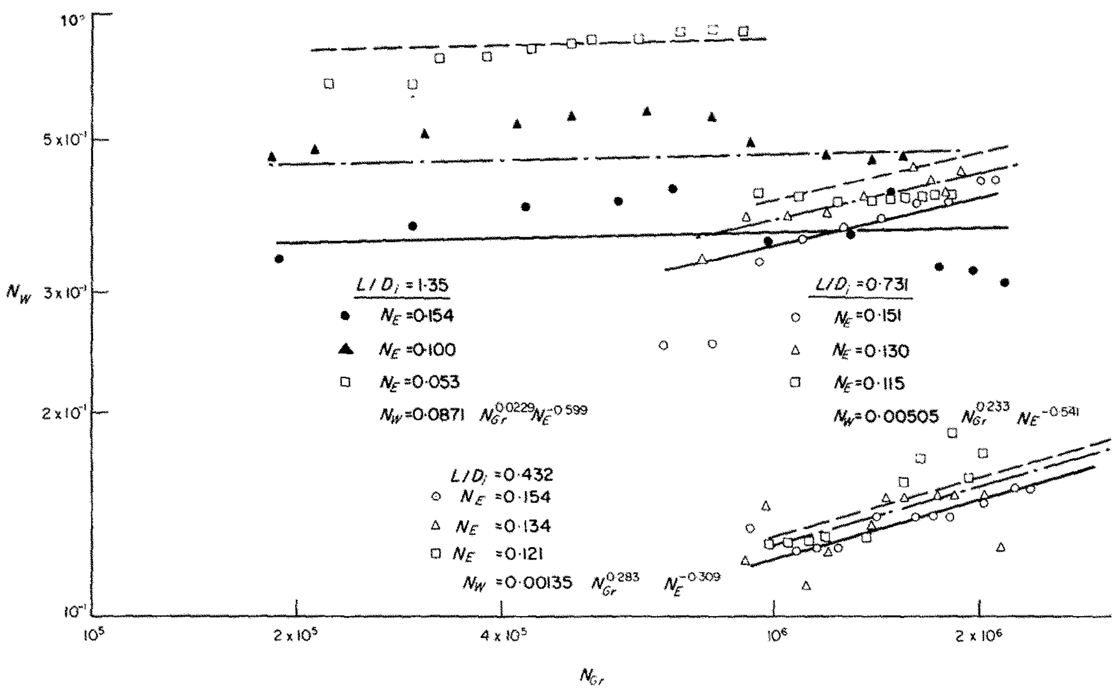

FIG. 7. Wavelength correlations.

width as an independent variable. Even then a single equation could not be found giving $N_W = f(N_E, N_{Gr}, N_D)$, but rather three separate equations, one for each relative gap width, resulted from an application of least squares techniques.

$$N_W = 0{\cdot}00135\, N_E^{-0{\cdot}309}\, N_{Gr}^{0{\cdot}283}, \quad \text{for, } L/2r_i = 0{\cdot}423. \tag{8}$$

$$N_W = 0{\cdot}00505\, N_E^{-0{\cdot}541}\, N_{Gr}^{0{\cdot}0223}, \quad \text{for, } L/2r_i = 0{\cdot}731. \tag{9}$$

$$N_W = 0{\cdot}0871\, N_E^{-0{\cdot}599}\, N_{Gr}^{0{\cdot}0229}, \quad \text{for } L/2r_i = 1{\cdot}35. \tag{10}$$

These equations produce average errors of $\pm 5{\cdot}5$, $\pm 7{\cdot}3$ and $\pm 6{\cdot}7$ per cent for relative gap widths of 0·423, 0·731 and 1·35 respectively. Over 90 per cent of the wave length data is represented by equations (8–10) with maximum deviations of less than ± 15 per cent. Due to the limited success in correlating the wave length data, this method is used primarily as a means of presenting the data and to enable the data to be approximately reproduced for comparison with future experimental and analytical work. In Fig. 7, wave length Reynolds numbers at three arbitrarily selected values of the expansion number for each relative gap thickness are plotted vs. the Grashof number. This was done for the sake of clarity and also to provide some indication of trends.

Techniques similar to those described above were again used in an attempt to correlate the amplitude of the oscillating flow, H_0. An amplitude Reynolds number was defined and a correlation was attempted using relative gap width,

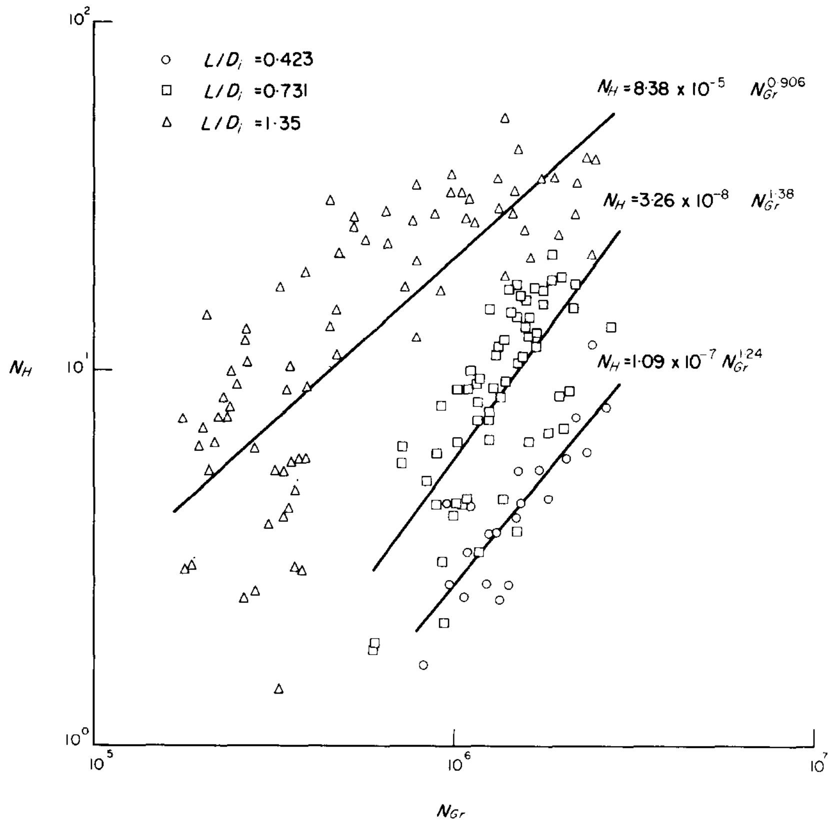

FIG. 8. Amplitude data.

Grashof number, and expansion number as independent variables. These efforts were unsatisfactory and a new search for dimensionless parameters with physical significance was conducted but without success. Finally, as a method of presenting the data only, the amplitude Reynolds number was correlated in terms of the Grashof number for each relative gap width. This data is presented in Fig. 8 along with equations for each relative gap width. These equations are given only to indicate the general trend of the data, however, a highly accurate amplitude correlation would not be expected since this quantity was extremely difficult to measure accurately.

The starting Grashof number, the value of the Grashof number at which the flow oscillations commence, was assumed to depend upon the expansion number and the relative gap width. The functional relation

$$(N_{Gr})_{\text{start}} = f(N_E, N_D)$$

was sought. It was observed taht plots of $(N_{Gr})_{\text{start}}$ vs. N_E using log–log coordinates gave a straight line for each N_D and showed that $(N_{Gr})_{\text{start}}$ has only a weak dependence on N_E and is thus primarily dependent on relative gap size. Neglecting N_E, a least squares technique yielded the following equation,

$$10^{-5} \times (N_{Gr})_{\text{start}} = 7{\cdot}68 - 1{\cdot}26\, N_D - 2{\cdot}56\, N_D^2, \qquad (11)$$

which correlates all of the data with maximum errors of ± 30 per cent. The maximum errors are for the runs made at the lowest ΔT's for which small experimental errors are magnified in the final determination both of $(N_{Gr})_{\text{start}}$ and N_E; 90 per cent of the data are represented by equation (11) with maximum errors of ± 7 per cent.

During the search for correlation equations it was found that the data could be correlated with a small percentage error by using some rather complex equations. The use of these equations was abandoned, however, in favor of the less accurate, but more physically significant, technique described. The correlation equations given in this paper for the wavelength, amplitude, and starting point data are used primarily as a means of presenting the data and not to claim significance for the equations themselves. Also, all the aforementioned equations are restricted to the range of operating conditions shown in Table 1. On the other hand, the correlation for the period provided by equation (7) coupled with the physical significance of the period Reynolds number certainly merits attention. This equation might prove particularly useful in verifying any stability analysis which might be put forth in the future.

A comparison of the results of this study with the work of other investigators strongly indicates that more work is needed on the effects of absolute cylinder dimensions on natural convective flow phenomena. For a gap thickness of about 1·27 cm and an inner cylinder diameter of about 16·51 cm, Liu, Mueller and Landis [6] reported a steady multicellular type flow pattern. For a gap thickness of about 2·03 cm and an inner cylinder diameter of about 6·1 cm, Grigull and Hauf [2] report an instability over a narrow range of Grashof numbers consisting of three-dimensional cells in the upper portion of the annulus only. In the present investigation neither of these types of flow patterns was observed for a gap thickness of about 1·78 cm and an inner cylinder diameter of about 4·06 cm. A detailed comparison of the apparatus used in this investigation with that of Liu, *et al.* [6] and that of Grigull and Hauff [2] reveals that the only significant difference in these three sets of equipment was the radius of curvature of the cylinders. Thus, it would appear that radius of curvature has a very strong influence on convective flow phenomena.

5. CONCLUSION

The characteristics of an oscillating flow condition in concentric cylindrical annuli were described, and measured values of the period, amplitude, and wave length were correlated in terms of dimensionless parameters obtained from a dimensional analysis of the problem.

Independent dimensionless groups were taken to be Grashof number, relative gap width, and expansion number, while period, amplitude, and wave length Reynolds numbers were defined as the dependent variables. A prediction equation, equation [11], was obtained that gives the Grashof number at which the oscillating flow will start. Equations (8–10) are shown to fit the wave length data reasonably well. Equation (7) provides an excellent correlation for the period data and should prove to be useful in future experimental and analytical studies.

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Résumé—Des renseignements quantitatifs détaillés sont présentés concernant les caractéristiques d'un écoulement oscillatoire de convection naturelle dans des conduites annulaires horizontales. Cet écoulement oscillatoire tridimensionnel inhabituel est décrit en donnant l'amplitude, la période et la longueur d'onde aussi bien que des résultats montrant le moment où ce type d'écoulement démarre à partir d'un écoulement préalablement stable à des nombres de Grashof plus faibles. La visualisation de l'écoulement d'air a été obtenue à l'aide de fumée de tabac. La pression dans l'espace annulaire variait de 13,8 à 275,8 kN/m^2 tandis que la différence de température entre les cylindres variait de 2,8°C à 55,5°C, ce qui donnait un nombre de Grashof (basé sur le diamètre du cylindre intérieur) allant de 290 à 2,7 × 10^6. On donne des équations de corrélation pour faciliter la présentation des données et permettre l'estimation du démarrage de cet écoulement oscillatoire ainsi que son amplitude, sa période et sa longueur d'ondre.

Zusammenfassung—Die Arbeit vermittelt eine ins Einzelne gehende, quantitative information über kennzeichnende Eigenschaften der oszillierenden natürlichen Konvektionsströmung in horizontalen, zylindrischen Ringräumen. Diese ungewöhnliche, drei dimensional oszillierende Strömung wird durch Angabe der Amplitude, Periode und Wellenlänge beschrieben, sowie durch Werte die angeben, wann die vorher vorhandene stabile Strömung bei kleineren Grashof-Zahlen, in die ozillierende umschlägt. Der Druck im Ringspalt wurde von 13,8 bis 275,8 kN/m^2 variiert; Die Temperaturunterschiede zwischen den Zylindern liessen sich von 2,8 bis 55,5 grd verändern, wobei sich für die Grashof-Zahl (mit dem Durchmesser des inneren Zylinders) ein Bereich von 290 bis 2,7 × 10^6 ergab. Korrelationsgleichungen erleichtern die Datenwiedergabe und erlauben eine Abschätzung des Oszillationsbeginns und die sich ergebende Amplitude, Periode und Wellenlänge.

Аннотация—Даются подробные количественные сведения о характеристиках колебательного течения при естественной конвекции в горизонтальных цилиндрических каналах. Это необычное трехмерное колебательное течение описывается данными об амплитуде, периоде и длине волны, а также свойствами стабильного потока, существовавшего при малых числах Грасгофа. Визуализация потока воздуха достигалась с помощью табачного дыма. Давление в каналах изменялось от 13,8 до 275,8 кн/м^2, в то время как температурная разность между цилиндрами изменялась от 2,8 до 55,5°C, причем число Грасгофа (рассчитанное по внутреннему диаметру цилиндра) изменялось от 290 до 2,7 × 10^6.

Приводятся корреляционные уравнения с тем, чтобы облегчить представление данных, а также определить возникновение колебательного течения, его амплитуду, период и длину волны.